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Introduction of Some Integral Transfroms

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ABSTRACT: This paper is defined the types of Integral Transforms. The generalized finite Mellin Integral Transforms in [0,a], which is mentioned in the paper by Derek Naylor and Lokenath Debnath. In this paper I also introduce Laplace Transform and Inverse Laplace Transform. The number method is used to find the time domain function from its frequency domain equivalent.

Keywords: Fourier Integral Transform; Laplace Transform and Mellin Transform.

INTRODUCTION: The Integral transforms play an important role in Analytic number theory, the part of Number theory where problems of a number-theoretic nature are solved by the use of various methods from Analysis. The most common integral transforms that are used are: Mellin transforms (Robert Hjalmar Mellin, 18541933), Laplace transforms (Pierre-Simon, marquis de Laplace, 1749-1827) and Fourier transforms (Joseph Fourier, 1768-1830). Crudely speaking, suppose that one has an integral transform

$$F(s) = \int_I f(t)k_1(s,t)\,dt \tag{1.1}$$

Where I is an interval, and K1(s,t) is a suitable kernel function. If a problem involving the initial function f(t) can be solved by means of the transforms F(s), then by the inverse transform

$$f(t) = \int_J F(s)k_2(s,t)\,ds \tag{1.2}$$

One can obtain information about f(t) itself. Here J is a suitable contour in $\mathbb{C}$ *and* $k_2(s,t)$ is another kernel function. Naturally the passage from (1.1) to (1.2) and back requires knowledge about the kernels, the convergence (existence) of the integrals that are involved, etc.

Fourier Integral Transforms: It was the work of Cauchy that contained the exponential form of the Fourier Integral theorem as

$$f(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{k(x-y)} f(y)\,dy\,dk. \tag{2.1}$$

Cauchy's work also contained the following formula for function of the operator D

$$;\emptyset(D)f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \emptyset(ik)e^{ik(x-y)} f(y)\,dy\,dk. \tag{2.2}$$

The deep significance of the Fourier Integral Theorem was recognized by mathematicians and mathematical physicists of 19th and 20th centuries. Indeed, this theorem is regarded as one of the most fundamental results of modern mathematical analysis and has widespread physical & engineering applications.

Laplace Transforms: The Laplace Transform of $f(x)$ is:

$$L[f(x)] = F(s) = \int_{0-}^{\infty} e^{-sx} f(x)\,dx \quad (Rs > 0)$$

Then $L^{-1}[F(s)] = f(x)$ is the Inverse Laplace Transform. It is unique if e.g. f(x) is continuous. The Inverse Laplace Transform can be represented by a complex inversion integral transform is

$$f(x) = \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} e^{sx}F(s)\,ds \qquad (x > 0)$$

And f(x)=0 for x<0. Here γ is a real number such that the contour of integration lies in the region of convergence of F(s). The Laplace Transforms are practical in view of the fast decay factor e^{-sx} which usually ensures good convergence. For an account on Laplace Transform we refer the reader to G.Doetsch's classic works [5] and [6].

Laplace Transform in the Circle and Divisor problem: The circle and divisor problems represent two classical problems of Analytic Number Theory. Much work was done on them and the reader is referred e.g., to [12],[9].

The (Gauss) circle problem is the estimation of

$$P(x) = \sum\nolimits'_{n\le x} r(n) - \pi x + 1$$

Where x>0, $\sum'_{n\le x}$ means that the last term in the sum is to be halved if x is an integer and $r(n) = \sum_{n=a^2+b^2} 1$ denotes the number of representations of n (€ N) as a sum of two integer squares

The Dirichlet divisor problem is the estimation of

$$\Delta(x) = \sum'_{n\le x} d(n) - x(\log x + 2\gamma - 1) - \frac{1}{4}$$

Where x>0, $d(n) = \sum_{\frac{\delta}{n}} 1$ is the number of divisor of n and $\gamma = -\acute{\Gamma}(1) = 0.5772157..$ is Euler's constant. One of the main problems is to determine the value of constants α, β such that

$$\alpha = \inf\{a \mid P(x) \le x^a\}, \beta = inf\{b \mid \Delta(x) \le x^b\}.$$

It is know that

$$\frac{1}{4} \le \alpha \le \frac{131}{416} = 0.314903 \ldots, \frac{1}{4} \le \beta \le \frac{131}{416} = 0.314903.$$

It is also generally conjectured that $\alpha = \beta = \frac{1}{4}$ but this conjecture seems very difficult to prove.

Mellin Transform: Hjalmar Mellin (1854{1933, see [59] for a summary of his works) gave his name to the Mellin transform that associates to a function f (x) is defined over the positive real the complex function f*(s) where

$$f^*(s) = \int_0^\infty f(x) e^{s-1}\, dx$$

This method is prescribed for generating such transforms. The method is adopted for the GMIT IN [0,∞] for the infinite interval x>0 and the result is modified for a finite interval 0<x<a. we see the properties for GFMIT in [0,a] like linearity property, scaling property, power property and propositions for the functions $\frac{1}{x} f\left(\frac{1}{x}\right)$ and $(\log x) f(x)$.

Basic Results: The generalized Mellin Integral Transforms of a function f(x) in 0<x<∞ introduced by Integral

$$M_-^\infty[f(x), r] = F_-^\infty(r) = \int_0^\infty \left(x^{r-1} - \frac{a^{2r}}{x^{r-1}}\right) f(x)\, dx, \quad r > 0.$$

The inverse of this transform is

$$M_-^\infty[f(x), r] = F_-^{\infty^{-1}}(r) = f(x) = \frac{1}{2\pi i} \int_L x^{-r} M_-^\infty(r)\, dr$$

Where L is the line Re p=c and $M_-^\infty[f(x), r] = F_-^\infty(r)$ is regular function on the strip $|Re(p)| < \gamma$ with c< γ On the other hand, if the derivative of f(x) is prescribed at r=a, it is convenient define the associated Integral Transform by

$$M_-^\infty[f(x), r] = F_+^\infty(r) = \int_0^\infty \left(x^{r+1} + \frac{a^{2r}}{x^{r+1}}\right) f(x)\, dx, \quad r > 0.$$

The inverse of this transform is

$$M_-^{\infty^{-1}}[f(x), r] = F_+^{\infty^{-1}}(r) = f(x) = \frac{1}{2\pi i} \int_L x^{-r} M_+^\infty(r)\, dr$$

Where L is the line Re p=c and $M_-^{\infty^{-1}}[f(x), r] = F_+^{\infty^{-1}}(r)$ is regular function on the strip $|Re(p)| < \gamma$ with c < γ.

CONCLUSION: This paper thus, consisted of a brief overview of the Integral Transforms. The use of these transforms is to solve differential equations. The primary use of Laplace Transform to convert a time domain Function into its frequency domain equivalent was also discussed.

REFERENCES:

1. Mellers, B. A. (2000) Choice and the relative pleasure of consequences, *Psychological Bulletin*, *126*, 910-924. http://dx.doi.org/10.1037/0033-2909.126.6.910
2. Zhao, S., Grasmuck, S., & Martin, J. (2008) Identity construction on Facebook: Digital empowerment in anchored relationships. *Computers in Human Behavior*, *24*(5), 1816-1836. http://dx.doi.org/10.1016/j.chb.2008.02.012
3. Bernstein, T. M. (1965) *The careful writer: A modern guide to English usage* (2nd ed.). New York, NY: Atheneum.
4. Guignon, C. B. (1998) Existentialism. In E. Craig (Ed.), *Routledge encyclopedia of philosophy* (Vol. 3, pp. 493-502). London, England: Routledge.
5. Jordan, J. J. (2005) *Psychosocial effects of gifted programming* (Unpublished master's thesis). University of Saskatchewan, Saskatoon, Canada.
6. Berg, D. H. (2003) *Prospective leadership development in colleges and universities in Canada: Perceptions of leaders, educators and students* (Unpublished doctoral dissertation). University of Saskatchewan, Saskatoon, Canada.
7. Hiebert, R. W. (2006) *The education of children from poverty: A descriptive case study of a public school and a community school* (Doctoral disserta-

tion). Available from ProQuest Dissertation & Theses: Full Text (NR18185).

8. Herculano-Houzel, S., Collins, C. E., Wong, P., Kaas, J. H., & Lent, R. (2008). The basic nonuniformity of the cerebral cortex. *Proceedings of the National Academy of Sciences*, *105*, 12593-12598. http://dx.doi.org/10.1073/pnas.0805417105.